Tool Path Generation for 3+2-axis Machining
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Abstract
3+2-axis machining combines the flexibility of orientation offered by indexable five-axis machines while maintains the low cost and ease of programming offered by three-axis machines. The efficiency of 3+2-axis machining can approach and may even surpass that of 5-axis machining but it requires special considerations to be carried out successfully. 3+2-axis machining offers the capability to orient the tool, but it is not affected by the variations of speed during machining encountered in simultaneous 5-axis machining. The tool orientation is locked during cutting, which results in a constant feed rate and a consistent surface finish. Since the motion is only in the three linear axes, programming and verification is much simpler than 5-axis machining. 3+2-axis machining requires identifying regions where the surface properties do not vary significantly. These regions or patches can be machined separately using a different tool orientation and feed direction. 3+2-axis machining strategies can be performed on 3-axis machines with the addition of a rotary/tilt table or on indexible 5-axis machines. These machines are much less expensive than simultaneous 5-axis machines and do not require excessive training because the tool paths can be calculated using 3-axis methods and software. The availability and relatively low cost of these machines compared to dynamically adjustable machines provides the motivation of this work; to combine the flexibility of orientation offered by indexable 5-axis machines while maintaining the low cost and ease of programming offered by 3-axis machines.

This paper presents a 3+2-axis machining strategy that is competitive with current machining methods. The paper presents the methodology to determine the surface partitioning, the tool orientation and the tool path. Machining tests are presented, and compared with actual machining methods, such as 3- and 5-axis machining.

2. Related Work
Researchers have attempted to take advantage of five-axis methods without using the expensive 5-axis machines. The methods are commonly based on a fixed tool orientation, which is optimized for a region and not at every point as in simultaneous 5-axis machining. 3+2-axis machining strategies have generally focused on the subdivision of the surface and finding tool orientations for those regions. Ralph and Loftus [3], Suh et al. [4] [5] developed 3+2-axis machining strategies that can be carried out on 3-axis machines. Yet, the methods require extensive computation and in most of the cases the partitioning is not guaranteed to be optimal. A different approach that was not based on the partitioning of a surface was developed by Gray [2]. This method, called 3+2-axis Arc-Intersect Method (AIM), uses a tool orientation that is optimized for each tool pass and not for a specified region.
However, this method requires a large number of orientation changes, which increases considerably the machining time for indexible 5-axis machines. A previous work by the authors [7] followed the concept proposed by Chen et al. [6] and developed a patch-by-patch machining method for sculptured surfaces. Both methods identify regions with similar surface properties using clustering algorithms. Although these methods confirmed the effectiveness of 3+2-axis machining they showed some limitations that included the use of local parameters. To overcome this limitation, this work will replace the partitioning parameters with the radius obtained using the RBM method. For each contact point, the RBM method can provide information about the shape and the radius of curvature. This 5-axis strategy can also be used to determine the tool orientation and tool position, which will be implemented as well for 3+2-axis machining. The theory and implementation of this method will be presented in the following section.

3. Surface Partitioning

The surface partitioning process starts by extracting surface properties for a sample of points, which are referred to as “features”. A feature vector is defined to contain the set of surface properties that are significant indicators of the shape and spatial location of the sample point. The k-means clustering algorithm uses the feature vector to define the surface patches. Although this method can sub-divide a surface into patches, it does not yield the optimal number of patches that results in the smallest machining time. To calculate the optimal number of patches, the surface is divided into different number of patches and the machining time is calculated for each partitioning. The machining time is calculated for a range of patches selected by the user. Finally, the partitioning that results in the smallest machining time is selected for machining.

A feature vector is formed by the surface coordinates ($S$) and the surface normal vectors ($N$) at the sampled points. These surface properties are normalized before use. This feature vector is used as the input to the k-means clustering algorithm. Finally, the k-means clustering algorithm is used to divide the data set into a determined number of patches.

After partitioning a surface, a tool path is generated for each patch. This step requires the boundaries of each patch. The boundaries are defined using the Minimum Intra Class Distance (MICD) method, which calculates a distance (in a two dimensional $XY$-plane) for a set of sample points ($Sx_i$ and $Sy_i$) and a contact point ($cp$) is defined as

$$d_{\text{intra class}} = \sqrt{(m - cp)^t c^{-1} (m - cp)} \quad (1)$$

where $m$ is the mean and $c$ is the covariance of every patch.

The method to determine the patch boundaries starts by calculating the mean and covariance for each patch as shown in Figure 1a. The mean is a point in the middle of the given set and is shown as a dark point. The covariance is shown in terms of a region that represents all the points that fall within the first covariance around the centroid in the given surface patch. The region is shown graphically as an ellipse as described in section 4.1. The MICD for each contact point is calculated from the mean and the covariance for each patch. The contact point belongs to the patch with the smallest MICD. The explicit boundaries of each point are not calculated. This classification method is used during tool path planning explained in section 4.4.

4. Tool path generation

Once the boundaries of the patches are defined, tool paths must be generated. Each patch can be machined using a particular feed direction, tool orientation and side step distance. The following subsections present the methods developed to select the tool path parameters for 3+2-axis machining.
Figure 1 Boundary limits are defined using the MICD method

4.1 Feed direction and tool orientation

In 3+2-axis machining, the calculation of the effective radius of the tool is based on the feed direction and tool orientation. Larger effective radius means larger side step and shorter tool path length, which is reflected in time savings. Maximum gains in the machining time will depend on the selection of optimal feed direction and tool orientation for each patch.

A method to calculate the feed direction and tool orientation was developed using the distribution of the surface normal vectors in a projected plane. First, the surface normals are calculated and are moved to the origin as illustrated in Figure 2a. The surface normals are projected on a 2-D plane and the tip of the normal vectors are used to determine the surface normals projected plane (Figure 2b).

Figure 2 Surface normals projected plane

The tips of the projected vectors occupy an area that is approximated by an ellipse, which fits the data to within one sigma. The parameters of the ellipse are defined by the eigenvalues and eigenvectors of the covariance of the surface normal vectors ($N_x$ and $N_y$). The real number $\lambda$ is called an eigenvalue of $c$ (covariance of $N_x$ and $N_y$). Every nonzero vector $v$ satisfying the equation (2) is an eigenvector of $c$ associated with the eigenvalue $\lambda$ [10]. The ellipse can be generated using the parameters shown in Figure 3. The direction of the major axis is defined by the eigenvector corresponding to the largest eigenvalue. The centre of the ellipse is positioned using the mean of the $X$-$Y$ coordinates of the projected surface normal vectors.
The feed direction is determined using one of the two axes of the ellipse, as shown in Figure 4a. If the feed direction is determined by the minor axis ($v_2$), the deviation of the surface normals and the tool axis will be large. This deviation is indicative that a large number of points will be machined by a region of the tool that does not lie in the front, thus minimizing gains due to shape matching. Thus, the feed direction should be along the major axis ($v_1$).

The vector $v_1$ has positive and negative directions. To determine the optimal feed direction, the surface normals are divided using the minor axes of the ellipse ($v_2$ and $-\ v_2$). The optimal feed direction is determined by the side having more normal vectors as shown in Figure 4b. This feed direction is used to calculate the tool orientation and minimize the variations of the tool axis with respect to the surface normals.

The tool orientation is calculated using the projected surface normals plane used to determine the feed direction. The first step to calculate the tool orientation is to determine the most inclined normal vector ($N_{\text{max}}$) with respect to the feed direction ($F$), as shown in Figure 5. The projection of $N_{\text{max}}$ in $XY$-plane is then projected along the feed direction to obtain $N_{\text{max}}$ repositioned. Finally, the tool axis can be positioned safely, by tilting the normal vector $N_{\text{max}}$ by a known amount to determine the tool orientation using equation 3.

$$T = N_{\text{max}}^* \cos(\theta) - \frac{(N_{\text{max}}^* \times F) \times N_{\text{max}}^*}{(N_{\text{max}}^* \times F) \times N_{\text{max}}^*} \sin(\theta)$$

The inclination angle ($\theta$) beyond the envelope is currently user selected and can be further optimized. If the tool inclination angle is large, the gains offered by shape matching are minimized, whereas if the tool inclination is small gouging may occur. A small angle can be applied safely to concave surfaces. For convex surfaces the angle $\theta$ is selected to be zero for improved efficiency. However, if a patch has concave and convex regions, it is treated as a concave surface for the purpose of determining the tool orientation.
4.2 Tool positioning

Three different types of cutting tools can be considered for the proposed 3+2-axis machining method: ball nose, toroidal and flat end-mills. The tool position $t_p$ (bottom centre of the tool) is given by Equation (4), where $S$ correspond to the surface coordinates, $R_1$ is the radius of the tool and $R_2$ is the radius of the insert. The toroidal tool equation is convenient because it can model both the ball nose ($R_1 = 0, R_2 = R$) and flat end milling cutter ($R_1 = R, R_2 = 0$).

$$tp = S + R_2 N + \frac{N - (N \cdot T) \cdot T}{|N - (N \cdot T) \cdot T|} R_1$$  (4)

4.3 Side step distance

The side step distance is one of the key factors in 3+2-axis machining because of its impact on the actual machining time. Material is left between tool passes in the form of scallops because the tool geometry is not exactly matched to the surface geometry. The side step distance is defined by finding the shortest distance between passes so that the largest scallop height is equal to a user-specified tolerance.

The side step depends on radius of tool and the inclination between normal (N) of surface at contact point and the tool axis (T) orientation. Although the tool is in general a toroidal cutter it can be approximated by a ball nose of radius equal to the radius of curvature of the torus at the contact point given in equation (5). The side step is given in equation (6).

$$eff \_radius = R_1 + \frac{R_2}{\sqrt{1 - (N \cdot T)^2}}$$  (5)

$$side \_step = 2 \sqrt{2 \ast eff \_radius \ast \varepsilon}$$  (6)

where $\varepsilon$ is the scallop height.

The side step obtained in equation (6) is a more accurate estimate for 5-axis machining, where the effective radius is dependant of the deviation between the tool axis and the surface normal. Since the tool orientation is fixed for a patch, in 3+2-axis the tool axis and the surface normal are not always on the same plane with respect to the feed direction. Thus, in 3+2-axis machining it is required to consider the effect of machining with the side part of the tool.

For a better estimate of the side step in 3+2-axis machining, the effective radius has to be projected to the plane perpendicular to the feed direction. To calculate the projected effective radius the tool axis ($T$) is projected into the surface normal ($N$) and normalized as given in equation (7). The Z-coordinate of the projected tool axis is dropped as the feed direction is expressed only in the $XY$-plane. Once the Z coordinates is dropped, the vector is projected onto the unit feed direction to calculate the angle ($\alpha$) between them. This angle ($\alpha$) is given in equation (8). This inclination is used to project the side step distance onto the proper feed direction and is given in equation (9).

$$PT = (N \cdot T)T$$  (7)

$$\alpha = \cos^{-1}\left(\frac{PF - (PT \cdot k)k}{|PT - (PT \cdot k)k|}\right)$$  (8)

$$eff \_sidestep = side \_step \ast \cos\left(\frac{\pi}{2} - \alpha\right)$$  (9)

4.4 Tool path planning

Once the surface is partitioned it can be machined. To machine the surface the user specifies the tool path pattern, namely
zigzag or parallel, and the feed direction of the tool. In the zigzag path the tool moves back and forth along the feed direction contacting the surface at all times. In the parallel path the tool cuts the surface as it moves in the feed direction, but lifts up and moves rapidly when returning.

Once the tool path pattern, zigzag or parallel, is known the exact path for each patch is determined. To machine the first patch the tool path is started from one end of the whole surface. The first point of the tool path is calculated from one of the corners of the surface. The first contact point is evaluated to find if it belongs to patch one. The evaluation is done by calculating the MICD distance with respect to mean and covariance of all the patches. The mean represents the center of the patch and the covariance represents the shape of the patch. If the contact point belongs to the patch to be machined, it is stored in a table. If it does not belong to the patch it is skipped and the next contact point is calculated. The next contact point is located at a user specified distance from the current point in the feed direction. Other methods of determining the next contact point can also be used. Once a pass has finished the side step method is used to find the first point in the next pass and the process repeats until the entire surface is covered. Only those points belonging to the first patch are stored in the table and are already pre-ordered into parallel passes. The process continues until the entire surface is covered. Once the first patch is complete, the process is repeated for all the remaining patches.

4.5 Machining Time
Knowing the tool path and the feed rate the time required to machine each patch is calculated by adding the retraction time estimates to the computed cutting time. From observations of the machine used in this work, five seconds is added each time to account for time consumed when the tool changes orientation. Since the optimal partitioning that results in the shortest machining time is not known apriori, the estimated times are calculated for partitioning the surface into an increasing number of patches starting at one. The process is stopped when further partitioning results in increased machining time and the partition that results in the shortest time is chosen for machining.

5. Implementation and Machining Tests
Machining tests were conducted on a test surface to validate the 3+2-axis machining method. The tests were done on a Deckel Maho 5-axis machining center. Although this machine can move all the 5-axis simultaneously, it was treated as an indexible machine and each patch was machined using only three axes, X, Y and Z. The axes A and C were only used to set the inclination of each patch.

The machining tests were carried out on a Bezier surface. This surface has characteristics that are similar to those commonly found in the manufacturing industry. The surface was represented using a grid of 50x50 uniformly space points and the surface properties were calculated at the 2500 points and assembled into a feature vector.

The proposed 3+2-axis method was applied to the surface and it was partitioned into up to eight patches, as shown in Figure 6a. It was observed that for more than eight patches estimated time continued to grow and thus the partitioning process was not continued. The tests conducted have verified that the developed strategy can identify the optimal number of patches that provides the lowest machining time while satisfying the surface requirements. For completeness, the proposed 3+2-axis machining is compared next with other multi-axis machining strategies.
5.1 Comparison between 3+2-axis machining and other multi-axis machining methods

The machining time obtained with the 3+2-axis machining method is compared with other common techniques. The comparison is conducted using experimental cutting tests. The first experiment is carried out using a 3-axis machining strategy with a ball nose end mill. In the second experiment the same surface is machined using a simultaneous 5-axis method known as "Sturz" method, using a 3 degrees inclination angle of the tool with respect to the surface normal is used for tool positioning. Figure 7 shows the machining test photo for the test surface. The results obtained show that 3+2-axis machining of the sample Bezier surfaces can be done in less time than the 3- and 5-axis, as illustrated in Table 1. The machining time for the proposed 3+2-axis machining method is about 23% and 19% smaller than 3-axis and 5-axis machining, respectively.

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Machining Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>3-axis</td>
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</tr>
<tr>
<td>5-axis</td>
<td>11.13</td>
</tr>
<tr>
<td>3+2-axis (improved)</td>
<td>9.05</td>
</tr>
</tbody>
</table>

Table 1 Machining time and tool path length comparison.

6. Conclusions and future research

This work demonstrated that 3+2-axis machining can be an efficient and practical alternative for surface machining. Although this method showed improvements in machining time, the biggest advantage is the reduced investment in machine cost and operator training. The 3+2-axis machining method proposed is based on the partitioning of surfaces. The optimal number of partitions is difficult to determine because of two opposing effects. A large number of patches leads to a better match between the tool and the workpiece, but it also leads to many tool reorientations between patches. On the other hand, if the number of patches is small, the benefit of the method is not fully realized since the shape of the tool may vary greatly from that of the surface. Accordingly, a technique for selecting the optimum number of partition is presented in this paper.

Reference